Exploiting nonlinearities for performance improvements in non-stationary control problems

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Non-stationary control problems

Output tracking

Disturbance rejection

+

Exosystem

=

Output regulation problem

Controlled synchronization

Observer design

Model reduction

Nonlinear performance analysis

Nonlinear performance-based design

Non-stationary control problems: tools

Linear systems: **Stability: local=nonlocal;** **Performance: Frequency Domain, H_infty,**
Internal Model Principle

Non-stationary control problems: tools

Linear systems: **Stability: local=nonlocal;** **Performance: Frequency Domain, H_infty, Internal Model Principle**

Nonlinear systems: **equilibrium stability <> stability of a non-stationary trajectory**

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Stability

Convergent systems

Incremental stability

Contraction analysis

Non-stationary control problems: tools

Linear systems: **Stability: local=nonlocal;** **Performance: Frequency Domain, H_infty, Internal Model Principle**

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Stability	Convergent systems	Incremental stability	Contraction analysis
Performance	Input-to-state convergence	Incremental ISS	Incremental L2 gain

Non-stationary control problems: tools

Linear systems: **Stability: local=nonlocal;** **Performance: Frequency Domain, H_infty, Internal Model Principle**

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Stability

Convergent systems | **Incremental stability** | **Contraction analysis**

Performance

Input-to-state convergence | **Incremental ISS** | **Incremental L2 gain**

+ Incremental small-gain results

Interconnections

Incremental passivity | **Incremental feedback passivity**

Non-stationary control problems: tools

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Interconnections

Incremental passivity

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Steady-state

Center manifold theorem

Invariant manifolds (based on convergence)

Regulator equations

The output regulation problem: methodology

Existence of a solution with zero regulation error (under some steady-state control action)

Solvability

Regulator equations

The output regulation problem: methodology

Existence of a solution with zero regulation error (under some steady-state control action)	**Solvability**	**Regulator equations**
Steady-state action of the controller	**Controller design**	**Internal model design**

The output regulation problem: methodology

Existence of a solution with zero regulation error (under some steady-state control action)

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Controller design

Stabilizing action of the controller

(incrementally) Stabilizing controller design

The output regulation problem: methodology

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Steady-state action of the controller	**Controller design**	**Internal model design**
Stabilizing action of the controller		**(incrementally) Stabilizing controller design**
Tuning for improved performance	**Tuning**	**Controller tuning**

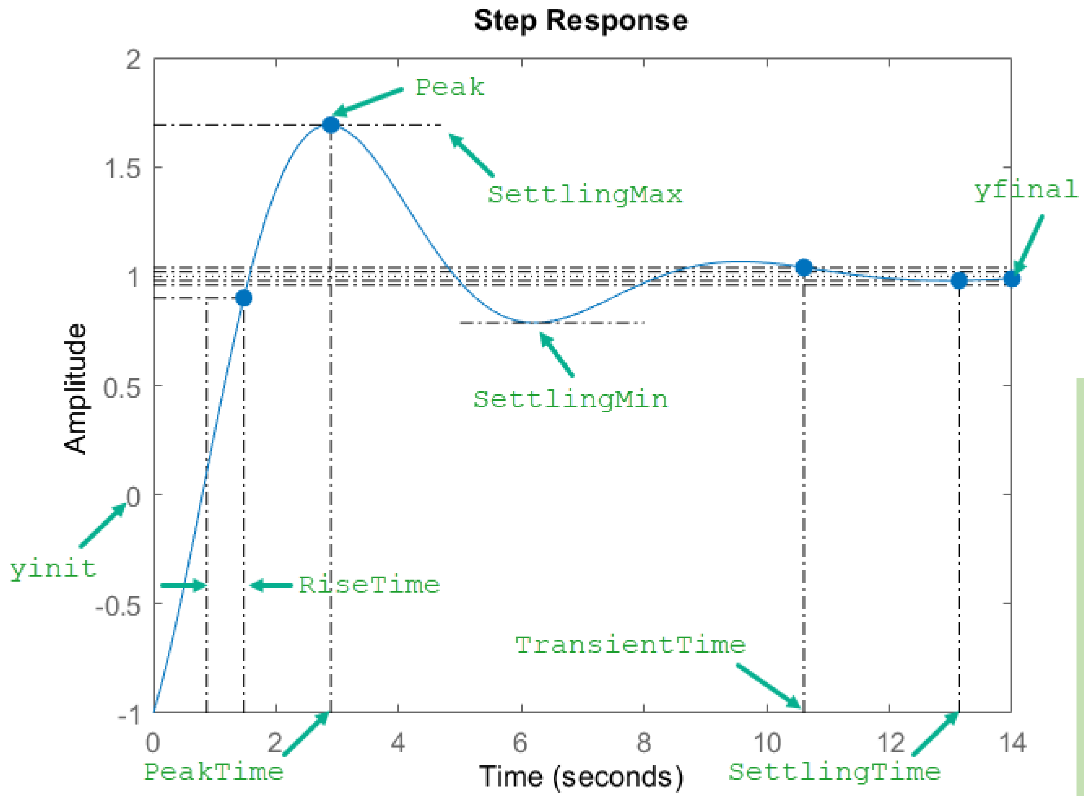
Controller performance

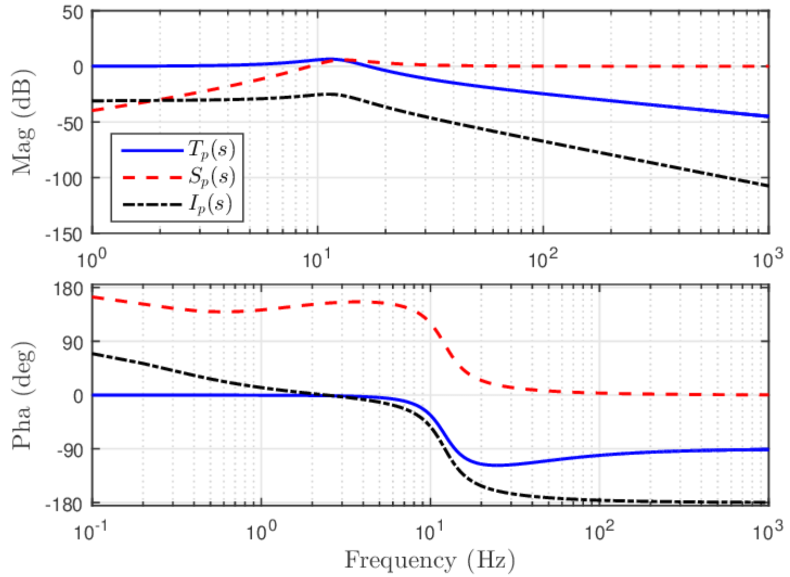
Current theory

- **Steady-state error**
- **Robustness**

Performance optimization

- **Rise time**
- **Overshoot**
- **Settling time**
- **Regulation error sensitivity to noise**
- **Control sensitivity to noise**
- **Energy efficiency of controller**





Tools for Linear systems

Controller performance

Current theory

Steady-state error

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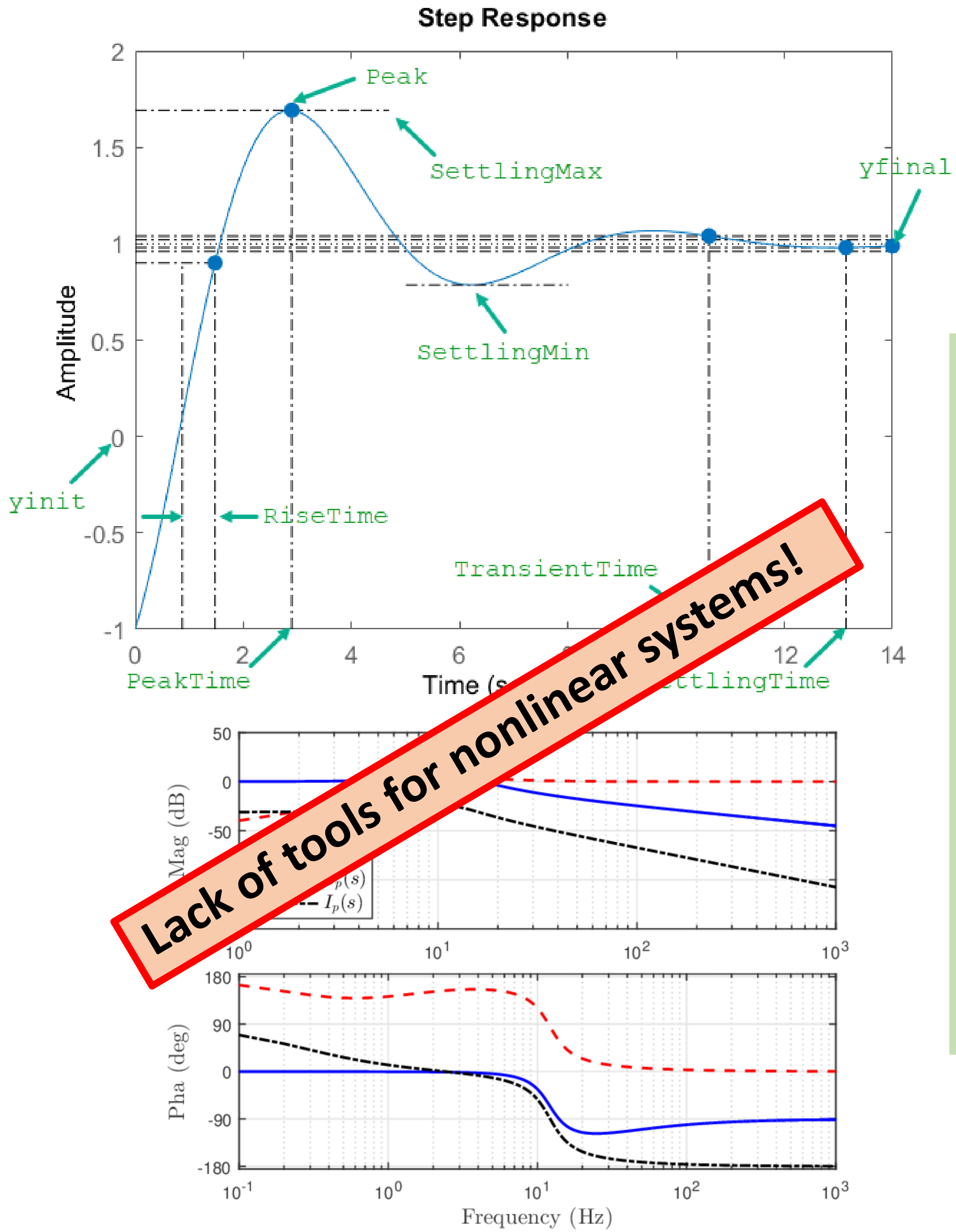
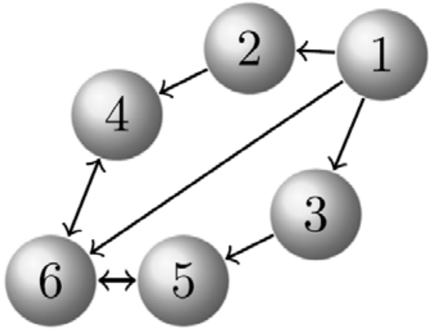
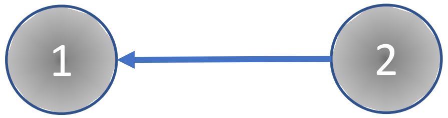


Tools for Linear systems

Controlled synchronization problem

Network of dynamic systems





System equations

u_i

$$\boxed{\begin{aligned} \dot{x}_i &= f(x_i, t) + Bu_i \\ y_i &= Cx_i \end{aligned}}$$

y_i

$$i = 1, \dots N$$

Control synchronization problem: find control laws for u_i such that

$$|x_i(t) - x_j(t)| \to 0, \quad \text{as} \quad t \to \infty, \quad \forall i, j$$

Constraints: 1) coupling law u_i can depend on y_i and y_j of neighboring system $j \in \mathcal{N}_i$

2) all solutions remain bounded for $t \geq 0$

3) in synchrony, the systems should exhibit the dynamics of $\dot{x}_i = f(x_i, t)$

Controlled synchronization problem

Net

HOW TO ACHIEVE SYNCHRONIZATION AND
IMPROVE PERFORMANCE OF THE COUPLING CONTROL LAWS?

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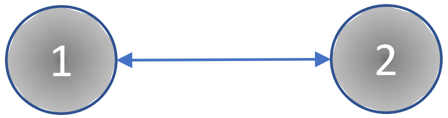
Motivating example

Synchronization of 2 systems:

$$\boxed{\dot{y}_i = y_i - \frac{1}{3}y_i^3 + g(t) + u_i}$$



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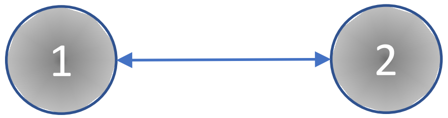

- Linear coupling: $\boxed{u_1 = -u_2 = \frac{1+\varepsilon}{2}(y_2 - y_1)}$

- Guaranteed convergence rate

$$|y_1(t) - y_2(t)| \leq e^{-\varepsilon(t-t_0)}|y_1(t_0) - y_2(t_0)|$$

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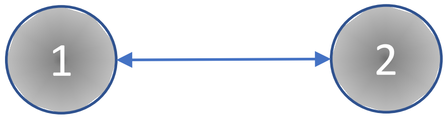

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- Nonlinear integral coupling:

$$\boxed{u_1 = -u_2 = \int_{y_1}^{y_2} \frac{\max\{1-s^2+\varepsilon, 0\}}{2} ds}$$

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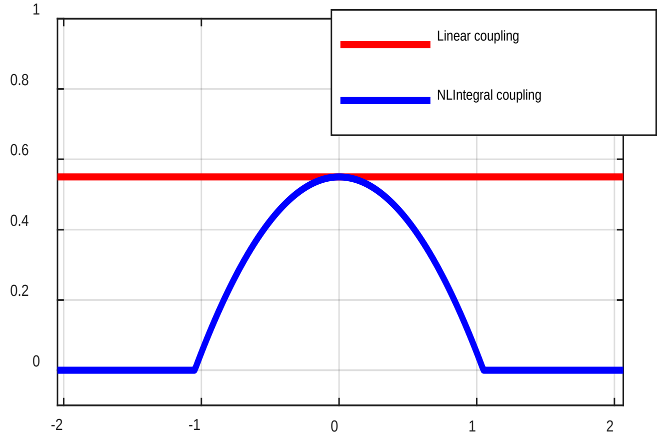

Coupling function

- Guaranteed convergence rate

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Motivating example

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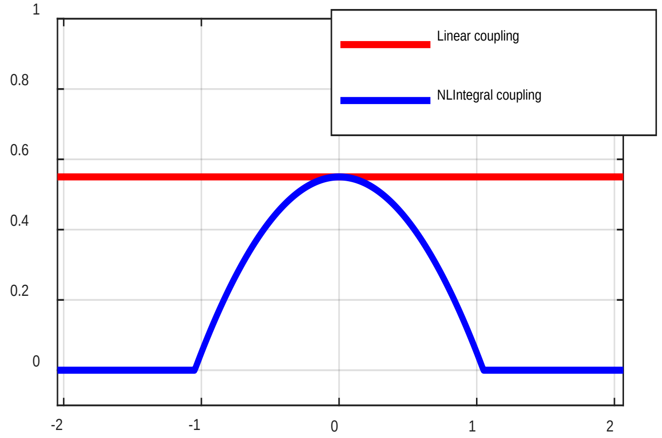


Coupling function

Synchronization is achieved/guaranteed with essentially lower coupling gains

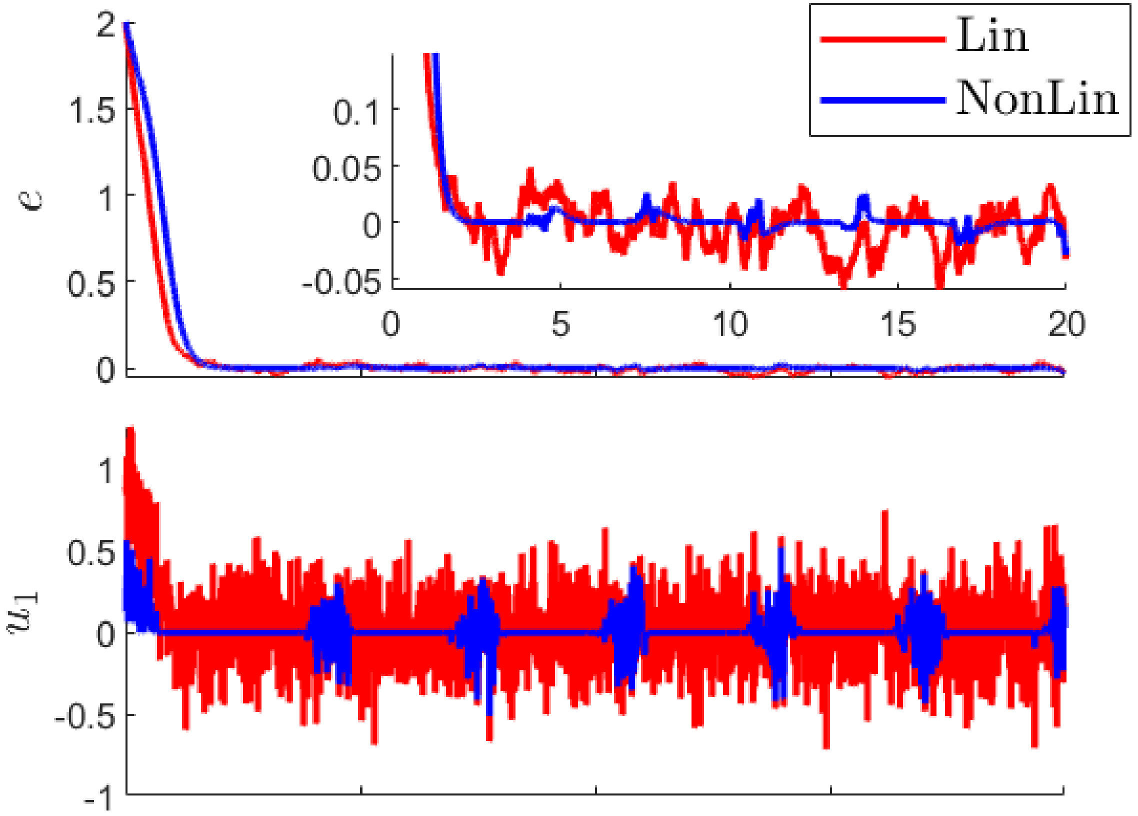
Motivating example

- **Lower coupling gains:**
  - Lower noise sensitivity
  - Lower energy needed for synchronization

$$||u_i^{Lin}||_{L_2} = 1.12$$

x3

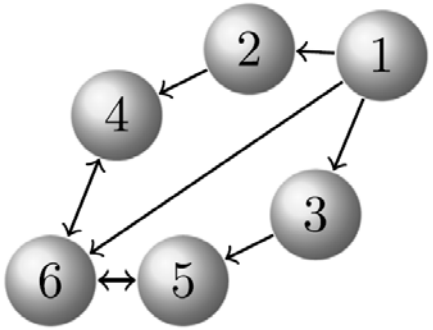
$$||u_i^{NLin}||_{L_2} = 0.35$$



Simulation results with measurement noise

Nonlinear integral coupling

Network of dynamic systems



System equations

$$u_i \rightarrow \boxed{\begin{aligned} \dot{x}_i &= f(x_i, t) + Bu_i \\ y_i &= Cx_i \end{aligned}} \rightarrow y_i$$

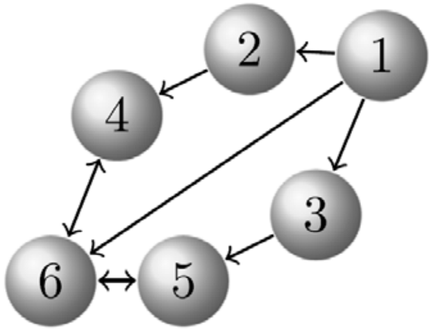
$$i = 1, \ldots N$$

Nonlinear integral coupling:

$$\boxed{u_i = \sum_{j \in \mathcal{N}_i} \int_{y_i}^{y_j} \lambda(s) ds}$$

Nonlinear integral coupling

Network of dynamic systems



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What are the conditions on minimal $\lambda(s)$ and the network graph to achieve synchronization ?

Nonlinear Integral Coupling – Design tools

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Bi-directionally connected systems

Theorem 1: If there exist $P = P^T > 0$, $R = R^T > 0$ and continuous function $\gamma(s)$ such that

$$P\frac{\partial f}{\partial x}(x) + \frac{\partial f^T}{\partial x}(x)P - 2C^T C\gamma(Cx) \leq -R, \quad PB = C^T, \quad \forall x \in \mathbb{R}^n$$

then the nonlinear integral feedback $u_1 = -u_2 = \frac{1}{2}\int_{y_1}^{y_2} \lambda(s)ds$ solves the controlled synchronization problem for any $\lambda(s)$ satisfying

$$\boxed{\lambda(s) \geq max(0, \gamma(s)), \ \forall s \in \mathbb{R}, \quad \int_{-\infty}^{+\infty} \lambda(s)ds \leq +\infty.}$$

Nonlinear Integral Coupling – Design tools

System equations

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Uni-directionally connected systems

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Nonlinear Integral coupling – Design tools

System equations

Uni-directionally connected systems

Incremental feedback passivity with a nonlinear gain function $\gamma(s)$

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Nonlinear Integral c

System equations

Incremental feedback passivity with a nonlinear gain function $\gamma(s)$

Definition 2. System (1) is called incrementally feedback passive with nonlinear gain $\gamma(s) \geq 0$ – denoted as $iFP(-\gamma(s))$ – if there exists a C^1 function $S(x) : \mathbb{R}^n \to \mathbb{R}_+$ and a function $\rho(x) : \mathbb{R}^n \to \mathbb{R}_+$ such that for any two inputs $u_a(t)$ and $u_b(t)$ any two solutions $x_a(t)$, $x_b(t)$ of system (1), for $N = 1$, corresponding to these inputs, with outputs y_a and y_b, satisfy the inequality

$$\frac{d}{dt}S(x_a(t) - x_b(t)) \leq -\rho(x_a - x_b) + (y_a - y_b)\left(\int_{y_b}^{y_a} \gamma(s)ds + u_a - u_b\right). \tag{5}$$

Theorem 1: If there exist $P = P^T > 0,\ R = R^T > 0$ and continuous function $\gamma(s)$ such that

$$P\frac{\partial f}{\partial x}(x) + \frac{\partial f^T}{\partial x}(x)P - 2C^T C\gamma(Cx) \leq -R, \quad PB = C^T, \quad \forall x \in \mathbb{R}^n$$

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Nonlinear Integral coupling

System equations

Ur

u_i $\quad \dot{x}_i = f(x_i, t) + Bu_i \quad y_i$

$y_i = Cx_i$

$i = 1, \dots N$

Constructive conditions are available for systems of the form

$$\dot{z} = q(z, y, t),\ \dot{y} = p(z, y, t) + u,$$
$$z \in \mathbb{R}^{n-1}, y, u \in \mathbb{R}, t \geq 0$$

Theorem 1: If there exist $P = P^T > 0,\ R = R^T > 0$ and continuous function $\gamma(s)$ such that

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$$\boxed{\lambda(s) \geq max(0, \gamma(s)),\ \forall s \in \mathbb{R}, \quad \int_{-\infty}^{+\infty} \lambda(s)ds \leq +\infty.}$$

Theorem *Consider system (★). Suppose there exist $(n-1)\times(n-1)$ matrices $Q = Q^T > 0$ and $M = M^T > 0$ such that inequality*

$$Q\frac{\partial q}{\partial z}(z,y,t) + \frac{\partial q^T}{\partial z}(z,y,t)Q \leq -M \tag{11}$$

holds for all (z,y) and $t \geq 0$. Let $\tilde{\gamma}(y)$ satisfy

$$\tilde{\gamma}(y) \geq \epsilon + \frac{\partial p}{\partial y} + \frac{1}{2}\left(Q\frac{\partial q}{\partial y} + \frac{\partial p^T}{\partial z}\right)^T (M - \epsilon I_{n-1})^{-1}\left(Q\frac{\partial q}{\partial y} + \frac{\partial p^T}{\partial z}\right) \tag{12}$$

for all (z,y) and some $\epsilon > 0$ satisfying

$$M - \epsilon I_{n-1} > 0, \tag{13}$$

where I_{n-1} is the $(n-1)\times(n-1)$ identity matrix. Then system (★) is iSFP$(-\gamma(s))$ with $\gamma(s) = \max\{0, \tilde{\gamma}(s)\}$ and quadratic positive definite $S(x)$ and $\rho(x)$

Constructive conditions are available for systems of the form

$$\dot{z} = q(z,y,t),\ \dot{y} = p(z,y,t) + u, \quad ★$$

$$z \in \mathbb{R}^{n-1}, y, u \in \mathbb{R}, t \geq 0$$

d continuous function $\gamma(s)$ such that

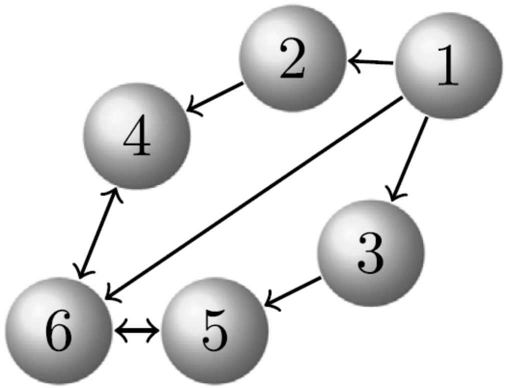
$$R, \quad PB = C^T, \quad \forall x \in \mathbb{R}^n$$

$\int_{y_1}^{y_2} \lambda(s)ds$ solves the controlled

$$\lambda(s) \geq max(0, \gamma(s)), \ \forall s \in \mathbb{R}, \quad \int_{-\infty}^{+\infty} \lambda(s)ds \leq +\infty.$$

Extension to larger networks: sequential decoloring of network graph

Conventional methods based on algebraic graph theory are not straightforward to apply



Incremental stability of closed-loop nodes (systems):

→Two nodes will synchronize if they are driven by the same number of asymptotically the same external inputs

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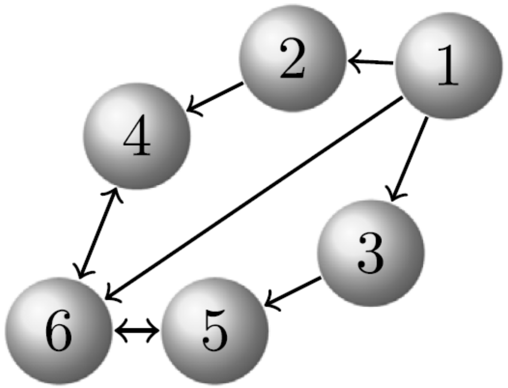

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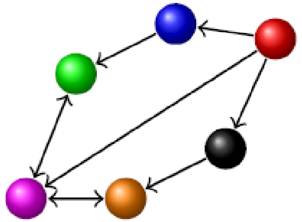

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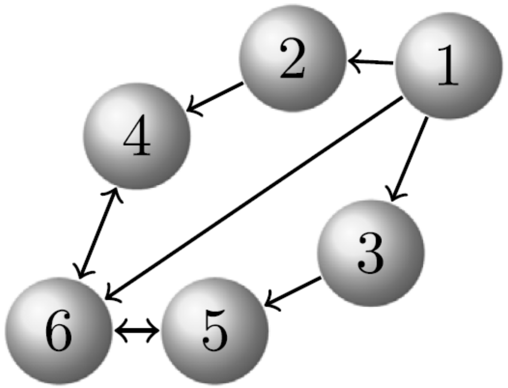

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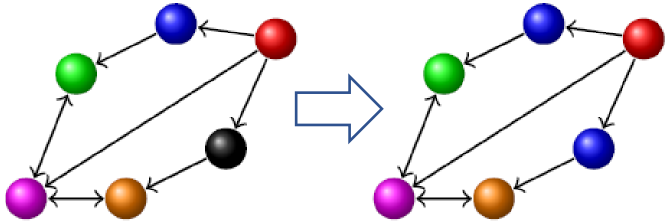

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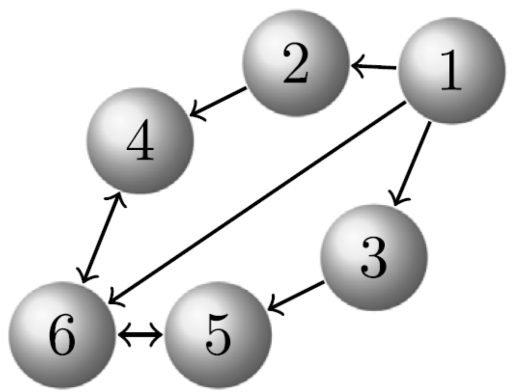

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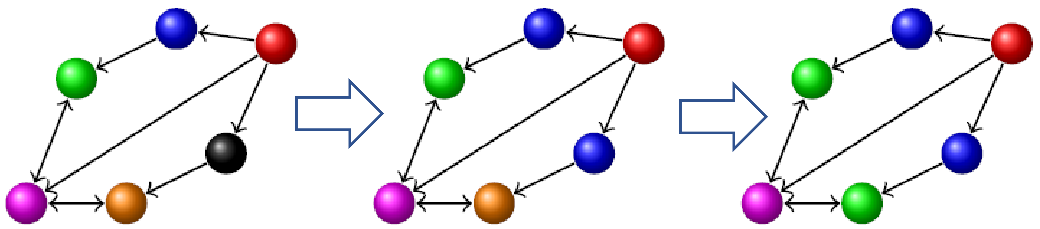

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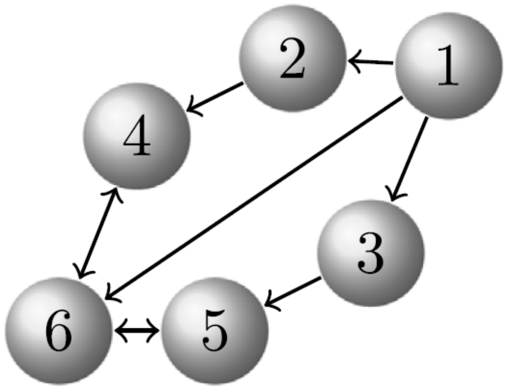
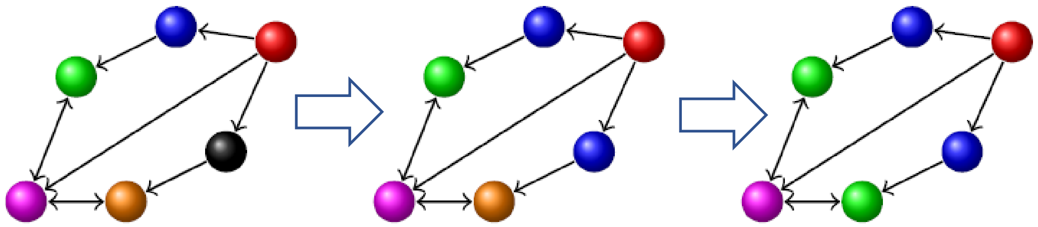


Incremental stability of closed-loop nodes (systems):

→Two nodes will synchronize if they are driven by the same number of asymptotically the same external inputs

→Two clusters of synchronizing nodes will synchronize with each other if each node in each cluster has the same number of asymptotically the same external inputs.



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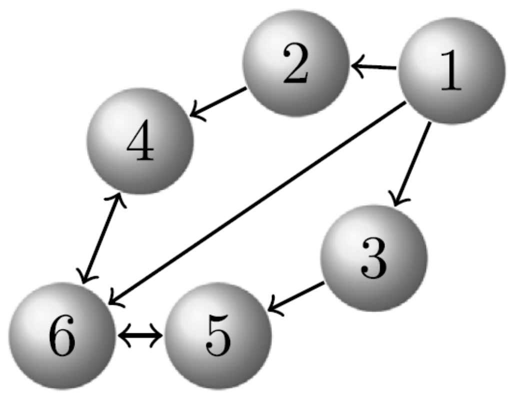

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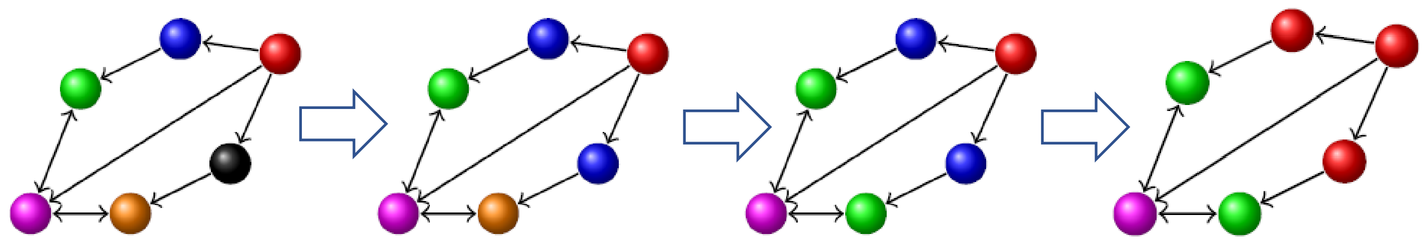

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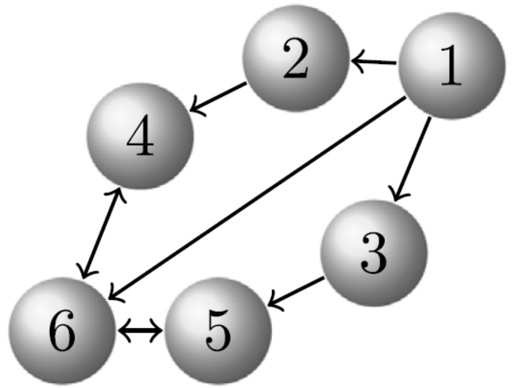

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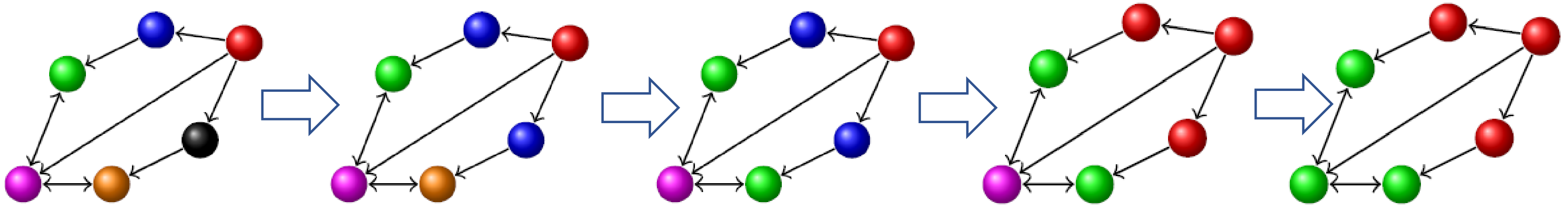
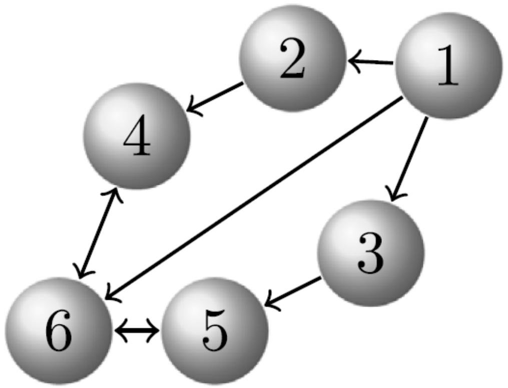
→Two clusters of synchronizing nodes will synchronize with each other if each node in each cluster has the same number of asymptotically the same external inputs.



Extension to larger networks: sequentional decoloring of network graph

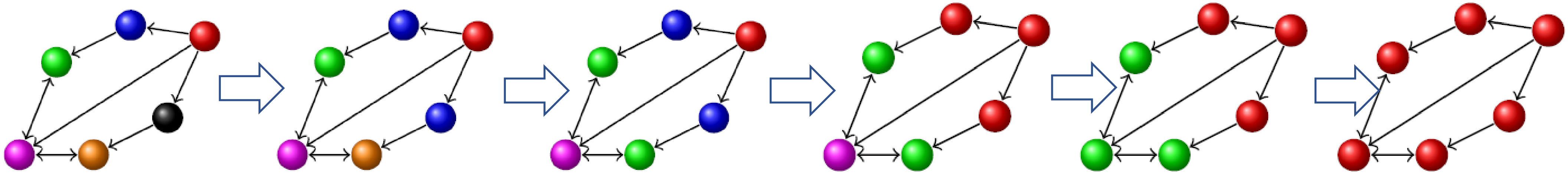
Conventional methods based on algebraic graph theory are not straightforward to apply



Incremental stability of closed-loop nodes (systems):

→Two nodes will synchronize if they are driven by the same number of asymptotically the same external inputs

→Two clusters of synchronizing nodes will synchronize with each other if each node in each cluster has the same number of asymptotically the same external inputs.



sequential decoloring of network graph

Extension to larger networks: sequentional decoloring of network graph

Conventional methods based on algebraic graph

to apply

Incremental sta

→Two nodes wi

same number of

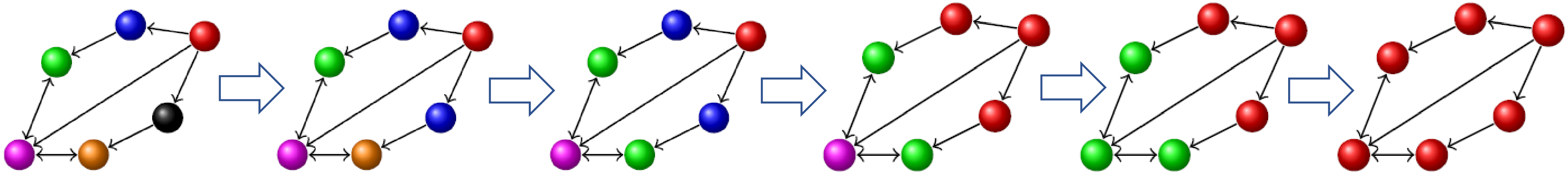
→Two clusters of

with each other if each node in each cl the same

number of asymptotically the same ext nputs.

Analysis of cluster synchronization

Algorithm to determine if a network graph is sequentially decolorable



sequential decoloring of network graph

Synchronization for larger networks

System equations

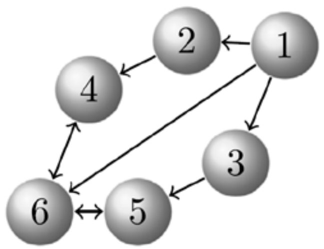
u_i

$$\boxed{\begin{aligned} \dot{x}_i &= f(x_i, t) + Bu_i \\ y_i &= Cx_i \end{aligned}}$$

y_i

$i = 1, \dots N$

Network graph



Theorem 2: **If the network graph is sequentially decolorable**, then for any coupling gain function $\lambda(s)$ from Theorem 1, the nonlinear integral coupling

$$u_i = \sum_{j \in \mathcal{N}_i} \int_{y_i}^{y_j} \lambda(s) ds$$

solves the controlled synchronization problem.

Example:
Synchronization of FitzHugh-Nagumo oscillators

System equations

$$\dot{z}_i = \frac{1}{\tau}\left(a + y_i - b z_i\right) + \frac{A_{ext}}{\tau}\cos\omega t$$
$$\dot{y}_i = y_i - \frac{1}{3} y_i^3 - z_i + r + u_i$$

Network graph

Integral coupling

$$u_1 = -u_2 = \frac{1}{2}\int_{y_1}^{y_2} \lambda(s) ds$$

$$\lambda(s) := \max\{0,\ \epsilon + 1 - s^2\}$$

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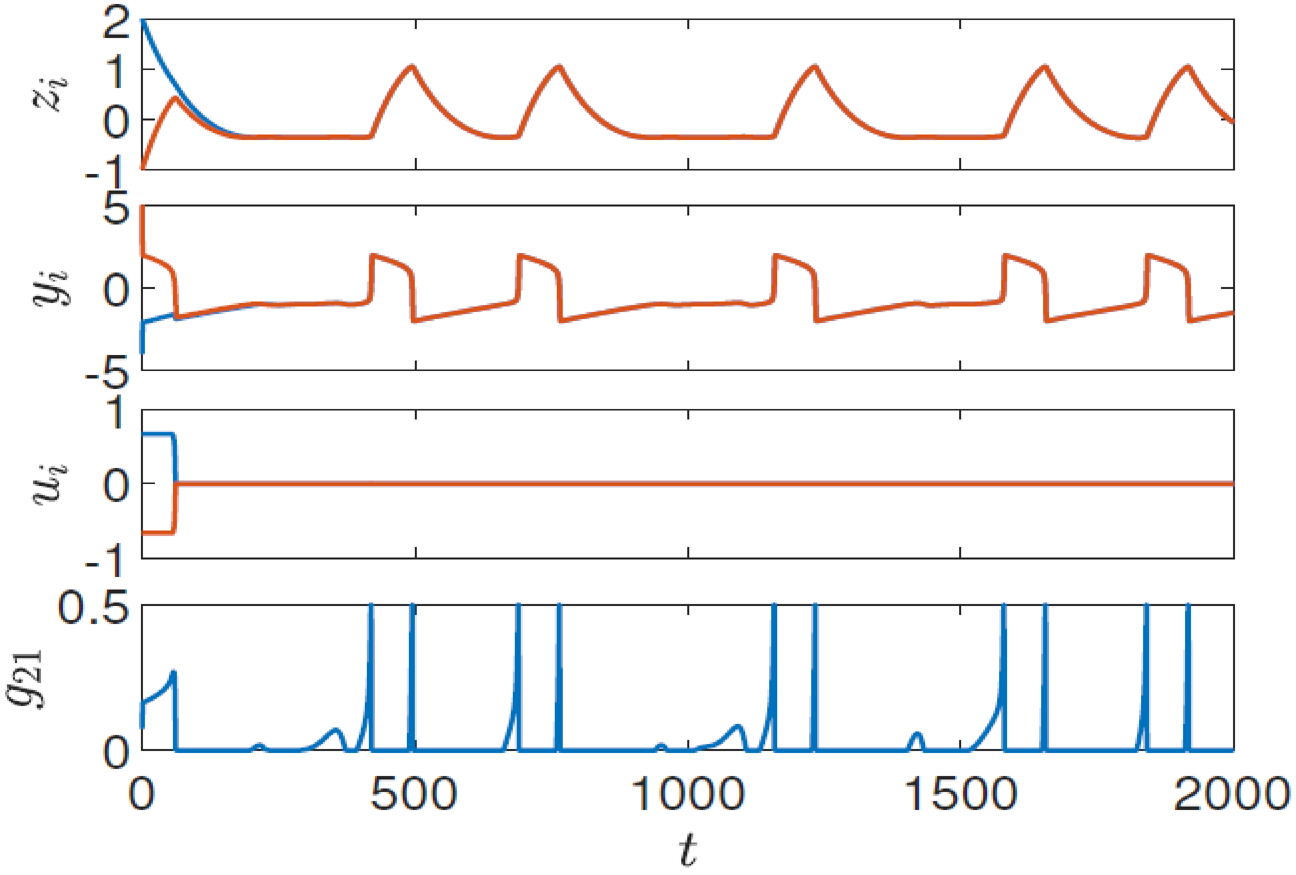


$$g_{21}(t) = \int_{y_2(t)}^{y_1(t)} \lambda(s)ds/(y_1(t) - y_2(t))$$

Example:
Synchronization of FitzHugh-Nagumo oscillators

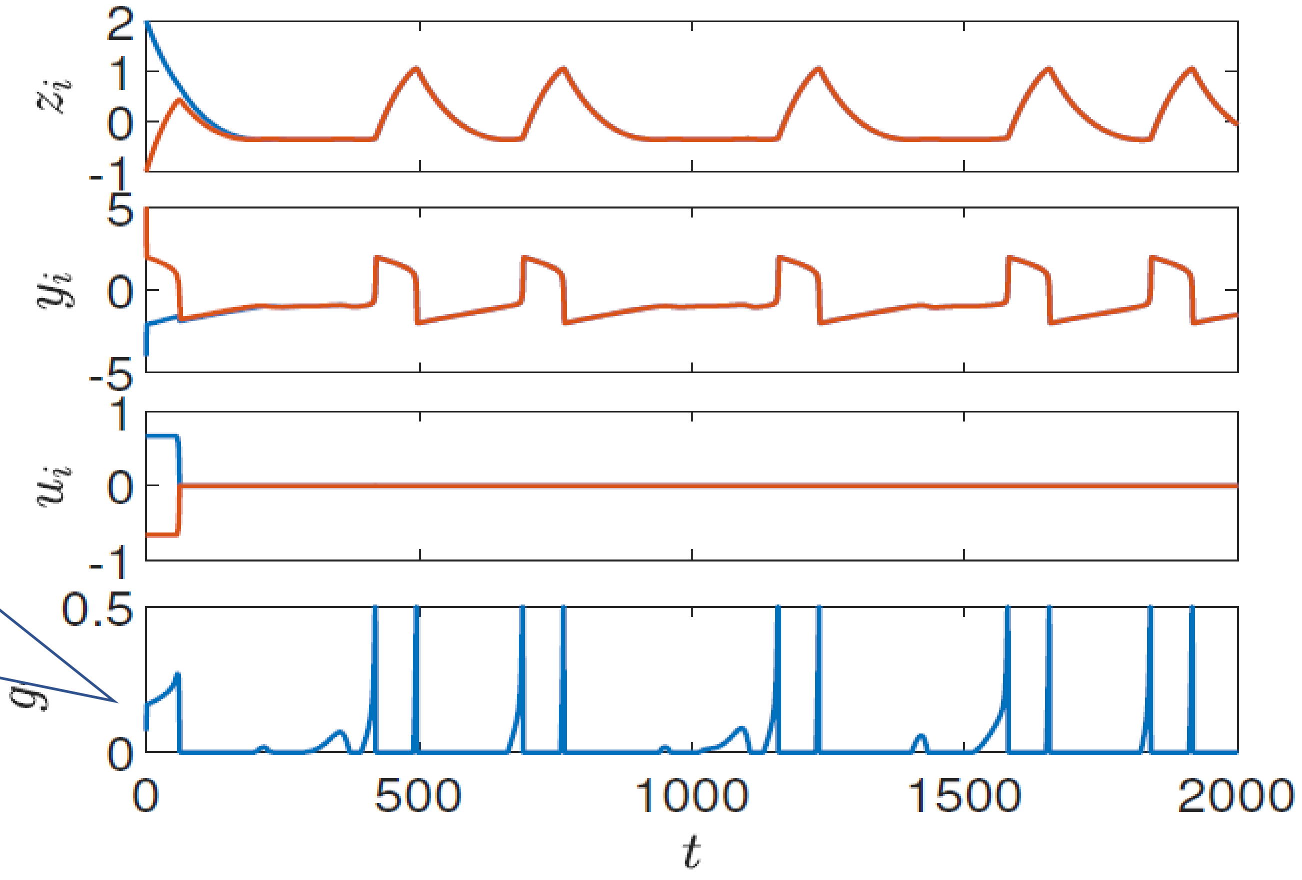
System equations

Synchronization is achieved through generically zero, but spiking coupling gain

Novel type of synchronization

$$u_1 = -u_2 = \frac{1}{2}\int_{y_1}^{y_2} \lambda(s)ds$$

$$\lambda(s) := \max\{0,\ \epsilon + 1 - s^2\}$$



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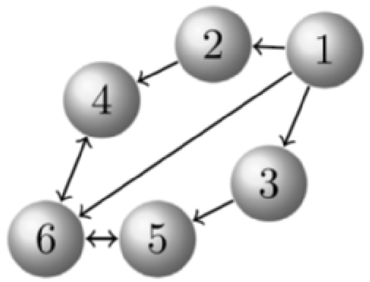
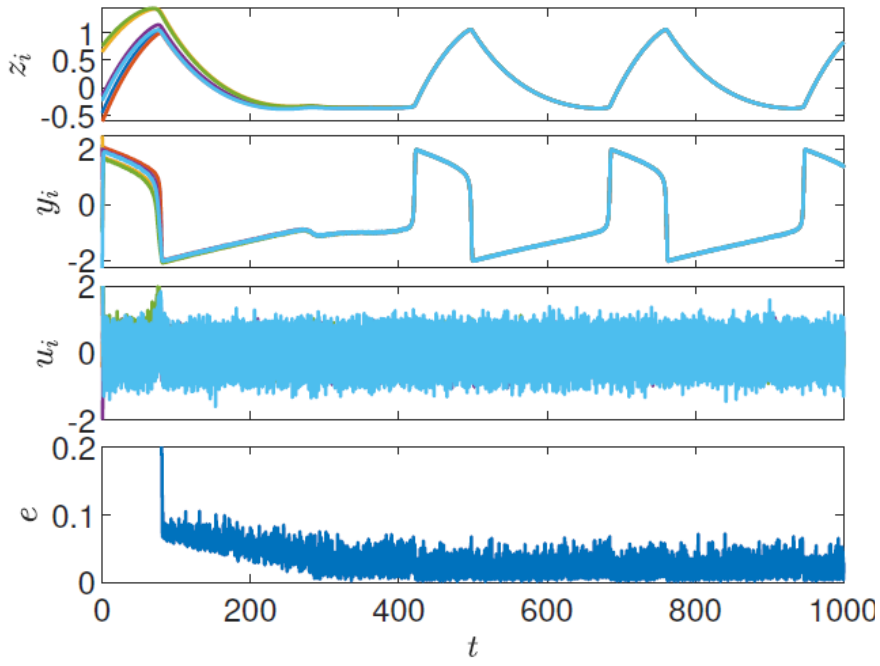
Example:
Synchronization of FitzHugh-Nagumo oscillators



+ measurement noise

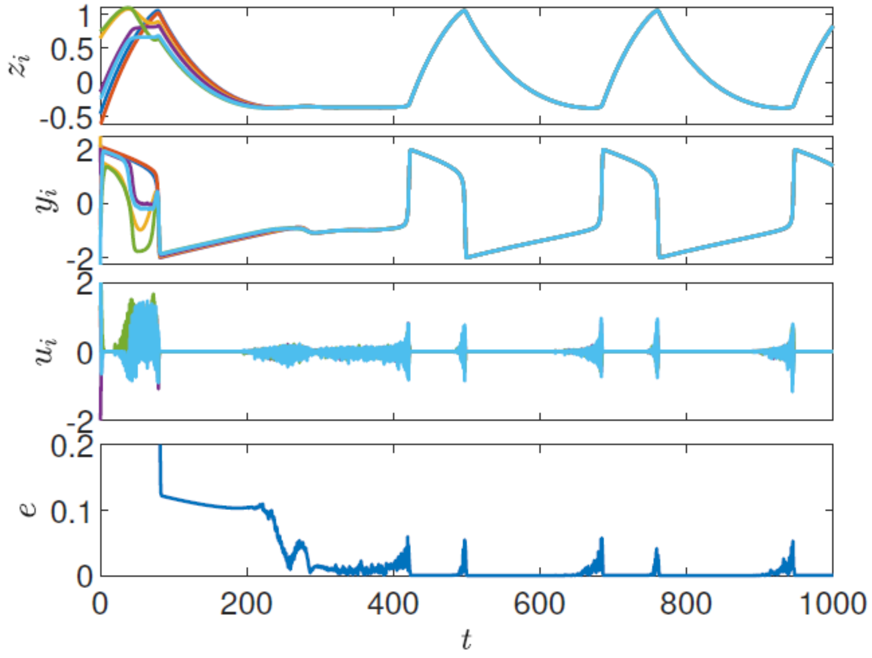
Linear coupling



$$||u^{lin}||_{L_2} = 18.9$$

$$||u^{lin}||_{L_\infty} = 10.0$$

Nonlinear integral coupling



$$||u^{nl}||_{L_2} = 10.6$$

$$||u^{nl}||_{L_\infty} = 2.7$$

Lower noise sensitivity

Lower energy for synchronization

Conclusions

- Nonlinear integral coupling – tool for performance improvement
  - Extra flexibility & benefits for synchronization
  - Lower sensitivity to measurement noise
  - Lower energy required for synchronization
- Constructive analysis and design tools for synchronization:
  - Results to design the nonlinear coupling function $\lambda(s)$
  - Sequential decoloring of network graph
- New type of synchronization of FitzHugh-Nagumo neuron models
  - Synchronization through spiking coupling gains

Conclusions

- Nonlinear integral coupling – tool for performance improvement
  - Extra flexibility & benefits for synchronization
  - Lower sensitivity to measurement noise
  - Lower energy required for synchronization

More details in

- A. Pavlov, E. Steur and N. van de Wouw, Nonlinear integral coupling for synchronization in networks of nonlinear systems, *Automatica*, 2022
- E. Steur, A. Pavlov, N. van de Wouw, Design of nonlinear coupling for efficient synchronization in networks of nonlinear systems, CDC 2023