Regulation without calibration

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Workshop on "Internal-model based regulation”
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Motivation :
A neurophysiological imitation game

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Mechanisms of Oscillation in Dynamic Clamp Constructed Two-Cell Half-Center Circuits

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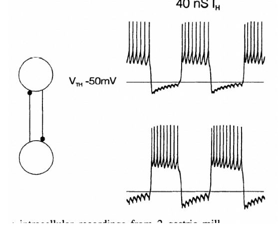
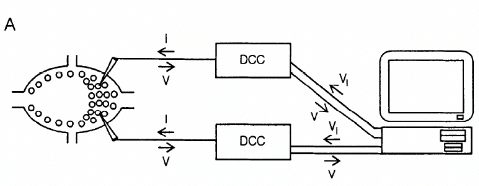
D



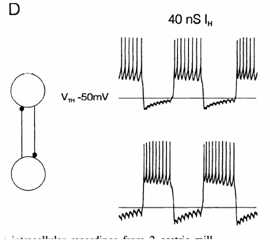
A



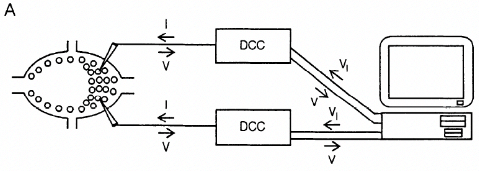
Question: how to emulate the biological oscillator D
with an artificial circuit replacing one of the two neurons (A) ?

The question:
how to regulate without calibration ?

D



A



The *calibration* solution:
Tune the parameters of the artificial neuron to minimise the mismatch between the natural and artificial behaviors.

Claim: the internal model principle *requires* calibration

Fact: this cannot work in practice because the biological neuron is highly variable.

Regulation without calibration

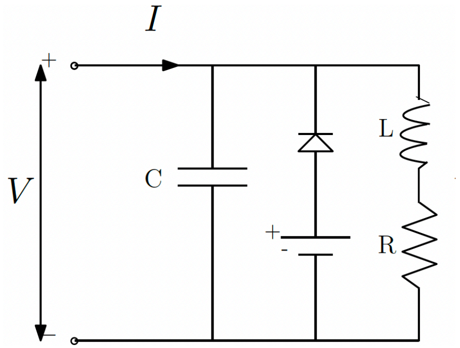
1. An academic example

2. Neuromorphic learning as a regulation problem

3. Lesson from the internal model principle : regulation requires calibration

4. Event-based regulation

Canonical example: Fitzhugh Nagumo circuit



$$
\begin{aligned}
C\dot{V} &= kV - \frac{V^3}{3} - I_L + I_{ext} \\
L\dot{I}_L &= -I_L + RV
\end{aligned}
$$

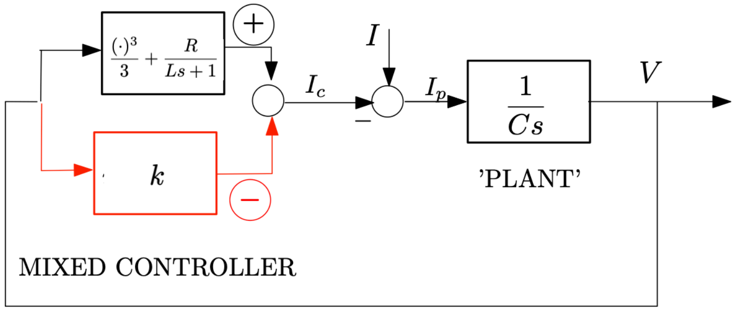
A circuit that reproduces the mechanism of nerve impulse:

R. FitzHugh, "Impulses and physiological states in theoretical models of nerve membrane," Biophysical journal, vol. 1, no. 6, p. 445, 1961.

J. Nagumo, S. Arimoto, and S. Yoshizawa, "An active pulse transmission line simulating nerve axon," Proceedings of the IRE, vol. 50, no. 10, pp. 2061–2070, 1962.

Referred to as "Bonhoeffer-van der Pol model" by FitzHugh after Van der Pol (1926).

The mixed feedback representation of Fitzhugh Nagumo circuit



The negative feedback circuit has fading memory
The positive feedback (or negative conductance) enables memory
The mixed feedback circuit has memory 'at scale'

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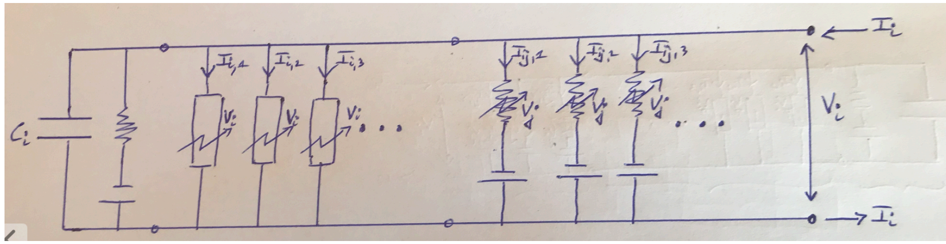
The circuit representation of a neuron



Leaky storage ("Passive membrane")

Internal current sources ("Ion channels")

External current sources ("synapses")

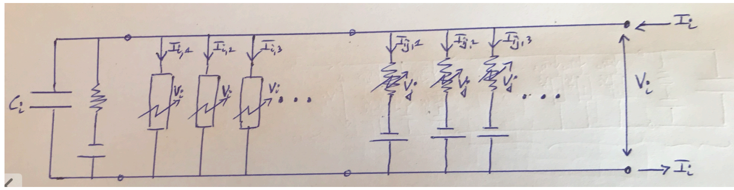
$$I_{i,s}(t) = w_{i,s} F_{i,s}(V_i \mid_{(-\infty,t)})$$

$$I_{ij,s}(t) = w_{ij,s} F_{ij,s}(V_j \mid_{(-\infty,t)})(V_i(t) - E_{ijs})$$

Each current source controls the circuit conductance in a specific temporal window

Tuning-modulating-learning the *weights* determines the circuit behavior

The circuit representation of a neuron

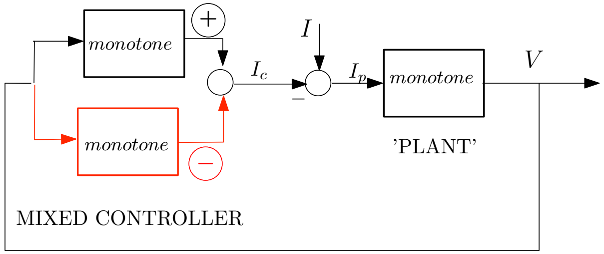


1. General *physical* representation of neural networks (including biophysical conductance-based models)

2. The role of *internal* conductances is often neglected relative to *external* (synaptic) conductances. It shouldn't.

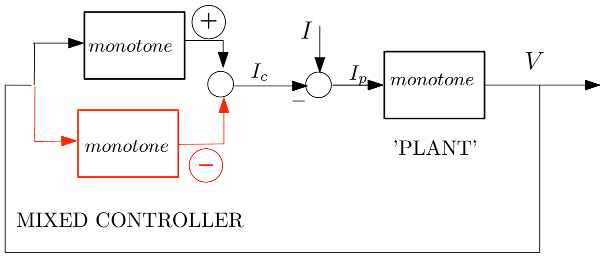
3. The restriction to *static* current sources corresponds to Hopfield Neural Nets. NNNs can be regarded as dynamical Hopfield Neural Nets.

The mixed feedback representation of a NNN



1. The "plant" represents the physical storage (RC network)
2. The "controller" is a parallel filter bank of voltage-gated current sources
3. "Monotone" = generalization of 'positive conductance' (Minty, 1960)

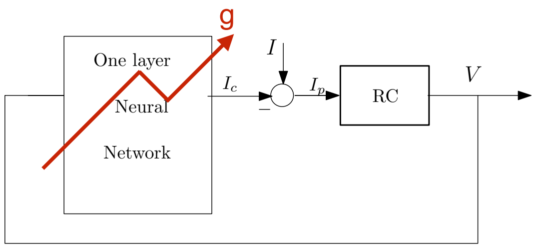
The significance of mixed feedback



1. Monotone circuits have *fading memory*; negative feedback preserves monotonicity
2. Positive feedback enables *memory*.
3. Mixed feedback enables *memory at scale.*

More meaningful than the traditional distinction between feedforward and recurrent networks.

Feedback machine learning



NNNs have the representation of a passive RC circuit in feedback with a one-layer feedforward layer.

The feedback loop is a substitute for many layers.

The important distinction is not between feedforward and recurrent, but rather between fading memory and memory.

NNNs provide a dynamic generalisation of Hopfield Neural Networks.

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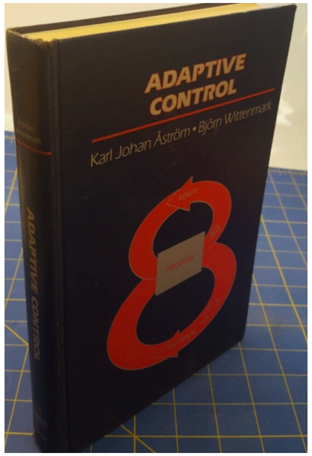
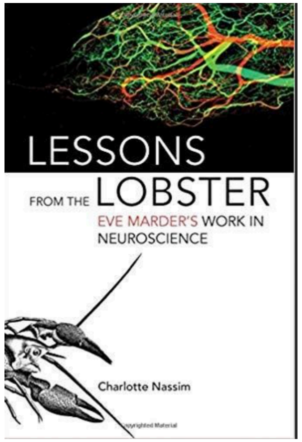
Neuromorphic Learning

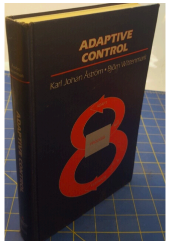
Neuromorphic learning = adaptive control = neuromodulation





50 years of research in engineering and in neuroscience to leverage from …

Adaptive Control



Chapter 1 What Is Adaptive Control?

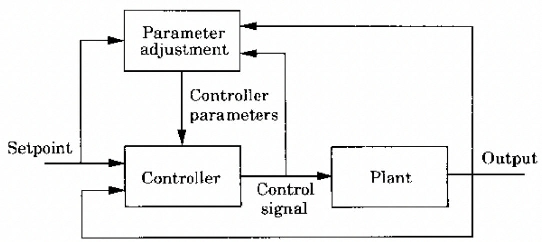
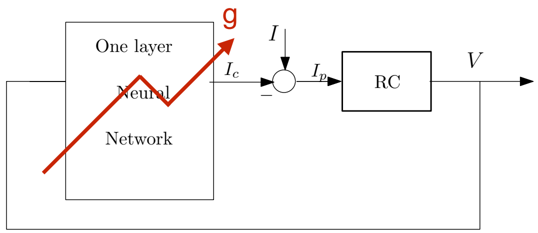


Figure 1.1 Block diagram of an adaptive system.

A theory developed in the 70s for linear systems.

The starting point :
Adaption (= Learning) is 'easy' under three conditions :
(i) linear parametrisation (ii) stable inverse (iii) relative degree one

NNNs are "easy" to adapt

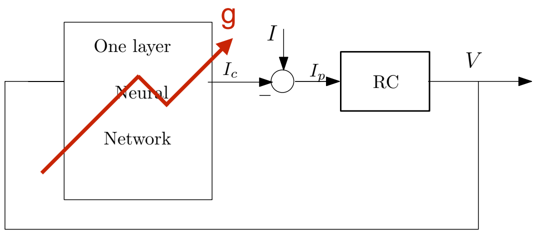


The starting point :
Adaption (= Learning) is 'easy' under three conditions :
(i) linear parametrisation : maximal conductances

(ii) stable inverse : I = difference of monotone (V)

(iii) relative degree one: RC has relative degree one !

Model reference Adaptive Control



Consider a reference trajectory $(I(\cdot), V_{ref}(\cdot))$
generated by a reference conductance g_{ref}

The learning rule is a linear regressor driven by the prediction error

$$e(t) = V_{ref}(t) - V(t)$$

A realm of learning rules

Recursive Least Squares estimation (RLS)

Least Mean Square estimation (LMS)

Stochastic gradient

MIT rule

Hebbian learning

…

All those learning rules proceed from (approximately) regressing the linear parameters from the residual error.

Simplifications rely on time-scale separation and distributed computation.

Some references

T.B. Burghi, R. Sepulchre. Online estimation of biophysical neural networks, to appear in IEEE Transactions on Automatic Control. (https://arxiv.org/abs/2111.02176)

T. B. Burghi, T. O'Leary, and R. Sepulchre. Distributed online estimation of biophysical neural networks. 61st IEEE Conference on Decision and Control, Cancun, Mexico, 2022.

R. Schmetterling, Th. Burghi, R. Sepulchre. Adaptive Conductance Control. Annual Reviews in Control Volume 54, 2022, Pages 352-3622

R. Schmetterling, Th. Burghi, R. Sepulchre. Robust Online Estimation of Biophysical Neural Circuits". 62nd IEEE Conference on Decision and Control, Singapore, 2023.

Current research questions :
robustness to uncertainty, scalability, circuit implementations

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Regulation as synchrony

Regulation can be defined as synchrony between a “reference behavior” and a “controlled behavior”.

Synchronisation is a general paradigm for regulation, adaptation, learning, observer design, …

Synchrony as contraction

Only one methodological framework to study synchronisation: *contraction* of the controlled behavior to the reference behavior.

The two conditions of regulation:

1. The error system is contractive
2. e=0 is solution of the closed-loop behavior

This is matching the exact original definition of regulation :

The Internal Model Principle of Control Theory*

B. A. FRANCIS† and W. M. WONHAM‡

The purpose of the compensator is twofold. First, it is to provide closed loop stability.

Second, it is to regulate a variable z which is a given function of the plant output c and the reference signal r; typically z may be the tracking error $r - c$. A plant-compensator combination with these two properties is termed a *synthesis*, and a synthesis is called *structurally stable* if these two properties are preserved when certain system parameters are perturbed.

The consequence of the IM principle:

Any design that proves regulation through regulation satisfies the internal model principle.

The internal model principle is a calibration theory: the internal model must be *calibrated* to the exosystem.

Calibration is not an artefact of our design methods. It is a necessary condition of robust synchronisation.

Where we stand with neuromorphic learning:

Neuromorphic neural networks belong to “easy” regulation problems

The challenge is *not* the regulation algorithm. Rather, the challenge is the robustness of the regulation algorithm to miscalibration.

Calibration is *necessary* but also *elusive* in variable environments.

How do animals regulate from sloppy internal models ?

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Event synchronisation without calibration

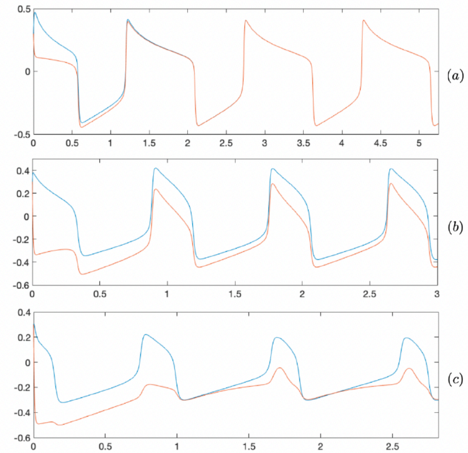

Fig. 1. Rapid synchronization of two identical (a) and non-identical (b) excitable systems under weak excitatory synaptic coupling. (c) Poor synchronization of the same non-identical excitable systems under strong diffusive coupling.

Rapid and robust synchronization via weak synaptic coupling, J.-G. Lee, RS, in press

Event synchronisation without calibration

Rapid and robust synchronization via weak synaptic coupling, J.-G. Lee, RS, in press

- Requires the combination of *excitable* nodes and *synaptic* coupling.
- Widely observed in biological neuronal populations
- First analysed for two neurons by Somers & Kopell (1993)
- Systems are nearly decoupled away from the events; “high-gain” coupling localised near the events

Towards an internal model principle for event-based systems

An internal model principle is necessary and sufficient for linear output synchronization
P. Wieland, R. Sepulchre, F. Allgöwer, Automatica, 2011

Phase synchronization through entrainment by a consensus input
P, Wieland, G. Schmidt, R. Sepulchre, IEEE CDC, 2010.

- No synchrony for the continuous-time dynamics, but “classical” regulatory design applied to “excitable” systems
- Synchrony (and the internal model principle) apply only to the discrete event map
- Hierarchy of events leads to hierarchy of regulation problems.
- A general solution for neuromorphic learning, consistent with what is observed in the animal world
- A control theoretic framework for event-based regulation theory.

Conclusions

1. Synchronisation is a general framework for nonlinear regulation
2. Neuromorphic learning belongs to the class of "easy" regulation problems
3. IM principle: regulation requires calibration
4. Bio-inspired solution for robust regulation: *event*-based regulation

THANK YOU !

Interested in more details ?

3 day course on *mixed feedback systems, Nov 29-Dec 1, Leuven, Belgium:*

https://sites.uclouvain.be/socn/drupal/socn/node/363